Review Notes. For Section X.56

Note. The following results will be useful to recall when covering Section X.56.

Example 15.39. The alternating group A_n is simple for $n \geq 5$. See also the online supplement to the Section 15 notes (`http://faculty.etsu.edu/gardnerr/4127/notes/An-is-Simple.pdf`).

Definition 35.12. A subnormal series $\{H_i\}$ of a group G is a *composition series* if all the factor groups H_{i+1}/H_i are simple.

Example. A composition series of S_5 is $\{\iota\} < A_5 < S_5$. The factor groups are A_5 and $\mathbb{Z}_2$. Notice that A_5 is nonabelian.

Definition 35.18. A group G is *solvable* if it has a composition series $\{H_i\}$ such that all factor groups H_{i+1}/H_i are abelian.

Example 35.19. Neither S_5 nor A_5 are solvable. In fact, A_5 is the smallest nonsolvable group.

Theorem 54.2. Let $s_1, s_2, \ldots, s_n$ be the elementary symmetric functions in the indeterminates $y_1, y_2, \ldots, y_n$. Then every symmetric function of $y_1, y_2, \ldots, y_n$ over F is a rational function of the elementary symmetric functions. Also, $F(y_1, y_2, \ldots, y_n)$ is a finite normal extension of degree $n!$ of $F(s_1, s_2, \ldots, s_n)$ and the Galois group of this extension is naturally isomorphic to S_n.

Theorem 56.4. Let F be a field of characteristic zero, and let $F \leq E \leq K \leq \overline{F}$, where E is a normal extension of F and K is an extension of F by radicals. Then $G(E/F)$ is a solvable group.

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