# Part I. Groups and Subgroups
## Study Guide

The following is a brief list of topics covered in Part I of Fraleigh's *A First Course in Abstract Algebra*, 7th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

**Section 0. Sets and Relations.**
Subset, empty set, improper and proper subsets, trivial subset, universal set, Natural Numbers $\mathbb{N}$, Integers $\mathbb{Z}$, Rational Numbers $\mathbb{Q}$, Real Numbers $\mathbb{R}$, Complex Numbers $\mathbb{C}$, Cartesian product of sets, relation between sets, function from one set to another, domain of a function, codomain of a function, range of a function, one to one (injection), onto (surjection), one-to-one correspondence (bijection), same cardinality of two sets, partition of a set, disjoint sets, cells of a partition, equivalence relation (reflexive, symmetric, transitive properties), partitioning a set into equivalence classes (Theorem 0.22), contrapositive.

**Section 1. Introduction and Examples.**
Complex numbers (real part, imaginary part, modulus or absolute value), Euler's Formula $e^{i\theta} = \cos\theta + i\sin\theta$, polar form of a complex number, argument of a complex number, $n$th roots of unity $U_n$.

**Section 2. Binary Operations.**
Binary operation, definition of "a set is closed under a binary operation," commutative binary operation, associative binary operation, tables for binary structures.

**Section 3. Isomorphic Binary Structures.**
Binary algebraic structure, isomorphism, show that two binary structures are isomorphic, structural property of a binary structure, identity element of a binary structure, uniqueness of an identity element (Theorem 3.13), isomorphisms map identities to identities (Theorem 3.14).

**Section 4. Groups.**
Definition of group, inverse element of a group element, general linear group of order $n$ $GL(n, \mathbb{R})$, abelian group, left and right cancellation laws in a group, uniqueness of identity and inverse elements in a group (Theorem 4.17).

**Section 5. Subgroups.**
Order of a group, subgroup of a group, improper subgroup, proper subgroup, trivial subgroup, nontrivial subgroup, showing a subset of a group is a subgroup (Theorem 5.14), cyclic subgroup of a group generated by an element, generators of a subgroup, cyclic group.

**Section 6. Cyclic Groups.**
Order of an element in a group, cyclic groups are abelian (Theorem 6.1), Division Algorithm, subgroup of a cyclic group is cyclic (Theorem 6.6), greatest common divisor, isomorphism of cyclic groups (Theorem 6.10), order of a subgroup of a cyclic group (Theorem 6.14).

**Section 7. Generating Sets and Cayley Digraphs.**
Klein 4-group, intersection of two subgroups is a subgroup (Theorem 7.4), subgroup generated by a set of elements of a group, Cayley digraph (generating and interpreting them), quaternions (as a finite group).

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