Part I. Groups and Subgroups

Study Guide

The following is a brief list of topics covered in Part I of Fraleigh’s *A First Course in Abstract Algebra*, 7th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 0. Sets and Relations.
Subset, empty set, improper and proper subsets, trivial subset, universal set, Natural Numbers \( \mathbb{N} \), Integers \( \mathbb{Z} \), Rational Numbers \( \mathbb{Q} \), Real Numbers \( \mathbb{R} \), Complex Numbers \( \mathbb{C} \), Cartesian product of sets, relation between sets, function from one set to another, domain of a function, codomain of a function, range of a function, one to one (injection), onto (surjection), one-to-one correspondence (bijection), same cardinality of two sets, partition of a set, disjoint sets, cells of a partition, equivalence relation (reflexive, symmetric, transitive properties), partitioning a set into equivalence classes (Theorem 0.22), contrapositive.

Section 1. Introduction and Examples.
Complex numbers (real part, imaginary part, modulus or absolute value), Euler’s Formula \( e^{i\theta} = \cos \theta + i \sin \theta \), polar form of a complex number, argument of a complex number, \( n \)th roots of unity \( U_n \).

Section 2. Binary Operations.
Binary operation, definition of “a set is closed under a binary operation,” commutative binary operation, associative binary operation, tables for binary structures.

Section 3. Isomorphic Binary Structures.
Binary algebraic structure, isomorphism, show that two binary structures are isomorphic, structural property of a binary structure, identity element of a binary structure, uniqueness of an identity element (Theorem 3.13), isomorphisms map identities to identities (Theorem 3.14).
Section 4. Groups.
Definition of group, inverse element of a group element, general linear group of order \( n \, GL(n, \mathbb{R}) \), abelian group, left and right cancellation laws in a group, uniqueness of identity and inverse elements in a group (Theorem 4.17).

Section 5. Subgroups.
Order of a group, subgroup of a group, improper subgroup, proper subgroup, trivial subgroup, nontrivial subgroup, showing a subset of a group is a subgroup (Theorem 5.14), cyclic subgroup of a group generated by an element, generators of a subgroup, cyclic group.

Section 6. Cyclic Groups.
Order of an element in a group, cyclic groups are abelian (Theorem 6.1), Division Algorithm, subgroup of a cyclic group is cyclic (Theorem 6.6), greatest common divisor, isomorphism of cyclic groups (Theorem 6.10), order of a subgroup of a cyclic group (Theorem 6.14).

Section 7. Generating Sets and Cayley Digraphs.
Klein 4-group, intersection of two subgroups is a subgroup (Theorem 7.4), subgroup generated by a set of elements of a group, Cayley digraph (generating and interpreting them), quaternions (as a finite group).

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