

Part II. Permutations, Cosets, and Direct Products

Study Guide

The following is a brief list of topics covered in Part II of Fraleigh's *A First Course in Abstract Algebra*, 7th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 8. Groups of Permutations.

Permutation, permutation composition/multiplication, computing products of permutations, S_n , D_n , labeling Cayley digraphs, image of a set, Cayley's Theorem.

Section 9. Orbits, Cycles, and the Alternating Groups.

Orbits of an element under a permutation, cycle, length of a cycle, cyclic notation, Theorem 9.8 (every permutation of a finite set is a product of disjoint cycles), transposition, writing a cycle as a product of transpositions, even permutations, odd permutations, A_n .

Section 10. Cosets and the Theorem of Lagrange.

Theorem 10.1 (which implies that the cosets of a subgroup induce an equivalence relation), left coset, right coset, sizes of cosets, Lagrange's Theorem, Corollary 10.11 (every prime order group is cyclic), Theorem 10.12 (the order of an element of a finite group divides the order of the group), the index of a subgroup in a group.

Section 11. Direct Products and Finitely Generated Abelian Groups.

Direct product (or sum) of groups, least common multiple, computation of the order of an element in a product of groups, Fundamental Theorem of Finitely Generated Abelian Groups, finding all abelian groups of a given order, decomposable/indecomposable groups, Theorem 11.16 (converse of Lagrange's Theorem for abelian groups), Theorem 11.17 (If m is a square free integer, then every abelian group of order m is cyclic).

Supplement. Small Groups.

Cyclic groups, dihedral groups, symmetry groups, alternating groups, direct products of groups, the Monster Group.

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