Part III. Homomorphisms and Factor Groups
Study Guide

The following is a brief list of topics covered in Part III of Fraleigh's *A First Course in Abstract Algebra*, 7th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 13. Homomorphisms.
Homomorphism, trivial homomorphism, projection mappings, image and inverse image of a set, Theorem 13.13 (properties of homomorphisms), kernel of a homomorphism, Theorem 13.15 and its meaning (the figure on page 6 of the notes), Corollary 13.18 (one to one homomorphisms in terms of kernels), normal subgroup.

Section 14. Factor Groups.
Factor group (or "quotient group"), Theorem 14.1 (how the factor group is defined and what it is isomorphic to), Theorem 14.4 (necessary and sufficient conditions for the definition of coset multiplication), Theorem 14.9 (relating normal subgroups to homomorphisms), The Fundamental Homomorphism Theorem, canonical isomorphism, canonical homomorphism, Theorem 14.13 (conditions equivalent to a subgroup being normal), automorphism, conjugation of element x by element g.

Section 15. Factor-Group Computations and Simple Groups.
$G/\{e\}$, G/G, S_n/A_n (Example 15.4), falsity of the converse of the Theorem of Lagrange (Example 15.6), Theorem 15.8 ($(H \times K)/\overline{H} \cong K$), classifying quotient groups using the Fundamental Theorem of Finitely Generated Abelian Groups, simple group, Theorem 15.15 (A_n is simple for $n \geq 5$), Theorem 15.16 (interaction of normal subgroups and homomorphisms), maximal normal subgroup, Theorem 15.18 (M is a maximal normal subgroup of G if and only if G/M is simple), center

of a group, commutator subgroup, Theorem 15.20 (the commutator subgroup of a group is a normal subgroup).

Supplement. The Alternating Groups An are Simple for $n \geq 5$.
The proof of Theorem 15.15 (or the solution to Exercise 15.39); A_2, A_3, and A_4 is not simple.

Supplement. Finite Simple Groups.
Cosets, normal subgroup, simple group, Jordan-Hölder Theorem, history of the result, papers and books about the result (by Daniel Gorenstein, Walter Feit, John Thompson, Peter Sylow, William Burnside, Leonard Dickson, John Conway), statement of the "Classification Theorem," Lie groups, the sporadic groups, The ATLAS.

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