

Part IV. Rings and Fields

Study Guide

The following is a brief list of topics covered in Part IV of Fraleigh's *A First Course in Abstract Algebra*, 7th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 18. Rings and Fields.

Definition of a ring, examples of rings, matrices made from ring elements, direct product of rings, arithmetic properties of a ring (Theorem 18.8), ring homomorphism, ring isomorphism, commutative ring, ring with unity, unit in a ring, division ring, field.

Section 19. Integral Domains.

Divisor of zero in a ring, the relationship between cancellation laws and zero divisors (Theorem 19.5), integral domain, be able to recognize the differences between rings/commutative rings/rings with unity/division rings/integral domains, \mathbb{Z}_p is a field, characteristic of a ring.

Section 20. Fermat's and Euler's Theorems.

Little Theorem of Fermat, use of Fermat's Theorem to calculate remainders, Euler phi-function, Euler's Theorem, use of Euler's Theorem to calculate remainders, solutions to equations $ax = b$ (Theorems 20.10 and 20.12).

Section 21. The Field of Quotients of an Integral Domain.

The 4 step method to build a field of quotients, the equivalence relation (Definition 21.1), the binary operations on F , the 9 field axioms, the relationship between the field of quotients F and integral domain D (Theorem 21.6).

Section 22. Rings of Polynomials.

Indeterminate, polynomial (infinite formal series, coefficients, and degree), ring of polynomials $R[x]$ (Theorem 22.2), algebraic numbers, rings of polynomials with two or more indeterminates, field of rational functions, evaluation homomorphisms (Theorem 22.4), zero of a polynomial, the basic goal.

Section 23. Factorization of Polynomials over a Field.

Division Algorithm (Theorem 23.1), Factor Theorem (Corollary 23.3), reducible and irreducible polynomials, factoring polynomials in fields, finding rational zeros (Corollary 23.12), Eisenstein Criterion, the p th cyclotomic polynomial, irreducible factors of a product of polynomials (Theorem 23.18), factoring polynomials into products of irreducible polynomials (Theorem 23.20).

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