Summary Notes. For Section X.51

Note. The following results are a summary of the "Greatest Hits" of Section X.51.

Theorem 51.6. If E is a finite extension of F, then $\{E : F\}$ divides $[E : F]$.

Definition 51.7. A finite extension E of F is a *separable extension field of* F if $\{E : F\} = [E : F]$. An element α of $\overline{F}$ is a *separable element over* F if $F(\alpha)$ is a separable extension of F. An irreducible polynomial $f(x) \in F[x]$ is a *separable polynomial over* F if every zero of $f(x)$ in $\overline{F}$ is separable over F.

Note 2. Now that we know $\{E : F\}$ divides $[E : F]$, we are interested in when these two quantities are equal. In this case, $\prod v_i = 1$ and each zero of $\text{irr}(\alpha_i, F(\alpha_1, \alpha_2, \ldots, \alpha_{i-1}))$ must be of multiplicity $v_i = 1$. So element α is a separable element over F if and only if $\text{irr}(\alpha, F)$ has all zeros of multiplicity 1.

Theorem 51.9. If K is a finite extension of E and E is a finite extension of F, that is $F \leq E \leq K$, then K is separable over F if and only if K is separable over E and E is separable over F.

Corollary 51.10. If E is a finite extension of F, then E is separable over F if and only if each $\alpha \in E$ is separable over F.

Definition 51.12. A field is *perfect* is every finite extension is a separable extension.

Theorem 51.13. Every field of characteristic zero is perfect.

Theorem 51.14. Every finite field is perfect.

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