

## Summary Notes. For Section X.53

**Note.** The following results are a summary of the “Greatest Hits” of the first half of Section X.53.

**Recall.** If  $F \leq E$ , the collection of all automorphisms of  $E$  leaving  $F$  fixed forms a group  $G(E/F)$ .

**Definition 51.7.** A finite extension  $E$  of  $F$  is a *separable extension field of  $F$*  if  $\{E : F\} = [E : F]$ .

**Definition 50.1.** A field  $E \leq \overline{F}$  is the *splitting field of  $\{f_i(x) \mid i \in I\}$  over  $F$*  if  $E$  is the smallest subfield of  $\overline{F}$  containing  $F$  and all the zeros in  $\overline{F}$  of each of the  $f_i(x)$  for  $i \in I$ . A field  $K \leq \overline{F}$  is a *splitting field over  $F$*  if it is the splitting field of some set of polynomials in  $F[x]$ .

**Definition 53.1.** A finite extension  $K$  of  $F$  is a *finite normal extension of  $F$*  if  $K$  is a separable splitting field over  $F$ .

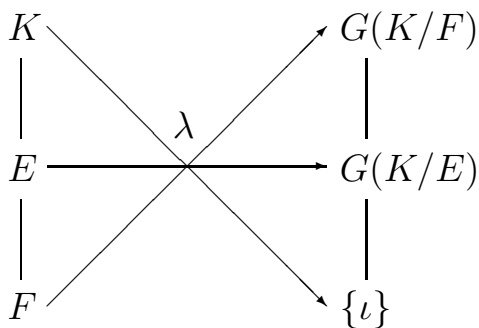
**Note.**  $K$  is a finite normal extension of  $F$  if and only if  $|G(K/F)| = \{K : F\} = [K : F]$ . (See the Note after Definition 53.)

**Theorem 48.11.** Let  $\{\sigma_i \mid i \in I\}$  be a collection of automorphisms of a field  $E$ . Then the set  $E_{\{\sigma_i\}}$  of all  $a \in E$  fixed by every  $\sigma_i$  for  $i \in I$  forms a subfield of  $E$ . (In particular, for  $F$  a subfield of  $K$  and  $H < G(K/F)$ ,  $K_H$  is the subfield of  $K$  left fixed by all automorphisms of  $K$  in  $H$ .)

**Note.** For a field  $E$ , where  $F \leq E \leq K$ , let  $\lambda(E)$  be the subgroup of  $G(K/F)$  leaving  $E$  fixed.

**Theorem 53.6. The Main Theorem of Galois Theory.** (Partial)

Let  $K$  be a finite normal extension of a field  $F$ , with Galois group  $G(K/F)$ . For a field  $E$ , where  $F \leq E \leq K$ , let  $\lambda(E)$  be the subgroup of  $G(K/F)$  leaving  $E$  fixed. Then  $\lambda$  is a one to one map of the set of all such intermediate fields  $E$  onto the set of all subgroups of  $G(K/F)$ :



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