Analysis 1+, MATH 4217/5217, Fall 2025

Homework 10, 5-2. Some Mean Value Theorems, 6-1. The Riemann Integral

Due Saturday, November 22, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the hypotheses, class notes, or textbook. Use the notation and techniques described in the in-class hints (this is part of the instructions!). Do not copy the work of others (including websites or AI generated solutions). If you have any questions, then contact me (`gardnerr@etsu.edu`).

5.2.3. Prove that if f is continuous on $[a,b]$ and if $f'(x) > 0$ on (a,b), then the minimum value of f on $[a,b]$ occurs at $x = a$ and the maximum value occurs at $x = b$.

6.1.2. Prove that if P and Q are partitions of $[a,b]$ such that Q is a refinement of P, then $\overline{S}(f;P) \geq \overline{S}(f;Q)$.

6.1.7. Prove Theorem 6-5: Suppose f is a Riemann integrable function on $[a,b]$. If I is a number such that $\underline{S}(f;P) \leq I \leq \overline{S}(f;P)$ for every partition P of $[a,b]$, then $I = \int_a^b f$.

6.1.A. Prove Theorem 6-9: The union of a countable collection of sets of measure zero is a set of measure zero. HINT: Think geometric series.