

Analysis 1+, MATH 4217/5217, Fall 2025

Homework 2, 1-2 Properties of the Real Numbers as an Ordered Field, 1-3 The Completeness Axiom

Due Saturday, August 6, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the hypotheses, class notes, or textbook. Use the notation and techniques described in the in-class hints (this is part of the instructions!). Do not copy the work of others (including websites or AI generated solutions). If you have any questions, then contact me (gardnerr@etsu.edu).

1.2.14. (a) Prove that $|a| - |b| \leq |a - b|$.

(b) Prove that $||a| - |b|| \leq |a - b|$.

1.2.23. Prove that if $a < b + \varepsilon$ for every $\varepsilon > 0$, then $a \leq b$.

1.3.4 (b) Between any two real numbers there is an irrational number. You may use the fact that $\sqrt{2}$ is irrational.

1.3.5 Prove that if A is a set of real numbers which is bounded below, then the greatest lower bound of A is unique.

1.2.A. (Graduate Problem) For $a, b \in \mathbb{R}$, $|a + b| = |a| + |b|$ if and only if $ab \geq 0$.