Analysis 1+, MATH 4217/5217, Fall 2025

Homework 3, 1-3 The Completeness Axiom, 2-1 Sequences of Real Numbers

Due Saturday, September 13, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the hypotheses, class notes, or textbook. Use the notation and techniques described in the in-class hints (this is part of the instructions!). Do not copy the work of others (including websites or AI generated solutions). If you have any questions, then contact me (`gardnerr@etsu.edu`).

1.3.11(a). The rational numbers are countable. HINT: You may assume that every nonzero rational number can be uniquely represented as $pq^{-1} = p/q$ where $p, q \in \mathbb{Z}$, p and q are relatively prime, $q \neq 0$, and $p > 0$.

1.3.11(b). Prove that the irrational numbers are uncountable.

2.1.9(a). Suppose that $\{a_n\}$ and $\{b_n\}$ are sequences with $\{a_n\} \to L$. If $\{a_n - b_n\} \to 0$, then $\{b_n\} \to L$.

2.1.9(b). If $\{a_n\}, \{b_n\}$, and $\{c_n\}$ are sequences with $a_n \leq b_n \leq c_n$ for every n and if $\{a_n\} \to L$ and $\{c_n\} \to L$, then $\{b_n\} \to L$. NOTE: The is the "Sandwich Theorem" for sequences.