Analysis 1+, MATH 4217/5217, Fall 2025

Homework 4, 2-1 Sequences of Real Numbers, 2-2 Subsequences

Due Saturday, September 20, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the hypotheses, class notes, or textbook. Use the notation and techniques described in the in-class hints (this is part of the instructions!). Do not copy the work of others (including websites or AI generated solutions). If you have any questions, then contact me (`gardnerr@etsu.edu`).

2.1.16. (b) Let $\{a_n\}$ and $\{b_n\}$ be sequences that diverge to ∞. Prove that $\{a_n + b_n\}$ diverges to ∞.

2.1.18. Prove that if $\{a_n\}$ is a sequence of nonnegative numbers and $\{a_n\} \to a$, then $\{\sqrt{a_n}\} \to \sqrt{a}$.

2.2.10 Any sequence $\{a_n\}$ that has the property $|a_{n+1} - a_n| < b^n$ for $b < 1$ is a Cauchy sequence. HINT: You may assume that $\{b_n\} \to 0$ (this follows from Exercise 2.2.2(c)).

2.1.8(b). (Graduate Problem.) Prove that if $\{a_n\} \to L$ and $a_n \leq K$ for every $n \in \mathbb{N}$, then $L \leq K$. Use the definition of limit.