Analysis 1+, MATH 4217/5217, Fall 2025

Homework 7, 4-1 Limits and Continuity

Due Saturday, October 25, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the hypotheses, class notes, or textbook. Use the notation and techniques described in the in-class hints (this is part of the instructions!). Do not copy the work of others (including websites or AI generated solutions). If you have any questions, then contact me (`gardnerr@etsu.edu`).

4.1.2. (d) (Slightly modified) Prove that the function $f(x) = x^2$ is continuous for $x > 0$.

4.1.14. Prove that $f(x) = 3x^3 + \sin x - 1$ has a zero between -1 and 1 (that is, a value of x between -1 and 1 where $f(x) = 0$). NOTE: You are not asked to *find* the zero, only to show that such a value exists. HINT: $\sin x$ is continuous on $\mathbb{R}$ by Exercise 4.1.13(a).

4.1.23. Suppose f is continuous on (a, b) and suppose that $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to b} f(x)$ exist. Prove that the function

$$F(x) = \begin{cases} \lim_{x\to a} f(x) & \text{if } x = a \\ f(x) & \text{if } x \in (a, b) \\ \lim_{x\to b} f(x) & \text{if } x = b \end{cases}$$

is uniformly continuous on $[a, b]$.

4.1.25. (c) Prove that the sum of two uniformly continuous functions is uniformly continuous.

4.1.27. (Graduate) Prove that if f is uniformly continuous and $\{x_n\} \in \mathcal{D}(f)$ is a Cauchy sequence, then $\{f(x_n)\}$ is a Cauchy sequence.