

Analysis 1+, MATH 4217/5217, Fall 2025

Homework 8, 4-2 Monotone and Inverse Functions

Due Saturday, November 1, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the hypotheses, class notes, or textbook. Use the notation and techniques described in the in-class hints (this is part of the instructions!). Do not copy the work of others (including websites or AI generated solutions). If you have any questions, then contact me (gardnerr@etsu.edu).

4.2.2. (d) Prove $\lim_{x \downarrow 0} \frac{1}{x} - \frac{1}{x^2} = -\infty$. HINT: Let K be an arbitrary number and consider the cases $K \geq 0$ and $K < 0$ separately. Notice what happens if $\delta \leq 1/2$.

4.2.9. (a) Prove that if $a > 0$ and $a \neq 1$, the function $f(x) = a^x$ is a one to one function.

4.2.9. (b) For $a > 0$ and $a \neq 1$, define $f(x) = \log_a x$ to be the inverse of a^x . Prove that $\log_a x$ is a continuous function.

4.2.11. Suppose that f is uniformly continuous on $[0, M)$ for all $M > 0$ and $\lim_{x \rightarrow \infty} f(x) = L$. Prove that f is uniformly continuous on $[0, \infty)$.