Analysis 1+, MATH 4217/5217, Fall 2025

Homework 9, 5-1 The Derivative of a Function

Due Saturday, November 8, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the hypotheses, class notes, or textbook. Use the notation and techniques described in the in-class hints (this is part of the instructions!). Do not copy the work of others (including websites or AI generated solutions). If you have any questions, then contact me (`gardnerr@etsu.edu`).

5.1.2(a) Use the definition of the derivative and the Binomial Theorem (Theorem 1-12), to show that the derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$ for any $n \in \mathbb{N}$.

5.1.3. In statistics it is important to know the value of x at which $f(x) = \sum_{i=1}^{n}(x - a_i)^2$ is minimized, where $a_1, a_2, \ldots, a_n$ are constants. Find this value, explaining the details of your differentiation. HINT: You may assume the First Derivative Test for Local Extrema (see my online note for Calculus 1 on Section 4.3. Monotonic Functions and the First Derivative Test and notice Theorem 4.3.A, or Exercise 5.2.4(a) in our next section).

5.1.10. Prove that $f(x) = \begin{cases} x^2 \sin(1/x) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$ is differentiable, but the derivative is not continuous at $x = 0$.

5.1.13. Suppose f and g have nth order derivatives on (a, b). Let $h(x) = f(x)g(x)$ for $x \in (a, b)$. Prove that for $x \in (a, b)$ we have $h^{(n)}(c) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(c) g^{(n-k)}(c)$. HINT: Pascal's Relation gives $\binom{\ell + 1}{k} = \binom{\ell}{k-1} + \binom{\ell}{k}$. See my online notes for Applied Combinatorics and Problem Solving (MATH 3340) on Section 1.2. Pascal's Triangle and notice Theorem 1.2.2.

5.1.12. (Graduate) Prove that

$$f(x) = \begin{cases} x \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous but not differentiable at $x = 0$.