INTRODUCTION.

NOTE. Mathematics is the study of ideas, not of numbers!!! This introduction to analysis class (Analysis 1, MATH 4217/5217) is primarily about the ideas of a real number, sets of real numbers, and functions of real numbers. We put on a rigorous foundation much of what you studied in the calculus sequence. In particular, we define the real numbers (as a complete ordered field), develop limits of sequences of numbers, develop limits of functions, study properties of continuous functions, and define derivatives. The topics of integration and sequences of functions can be found in the second analysis class (Analysis 2, MATH 4227/5227).

Notation. Some of the sets of numbers and standard symbols we will encounter include:

$\Rightarrow$ stands for "implies,"

$\Leftrightarrow$ stands for "if and only if" (abbreviated "iff"),

$\forall$ stands for "for all,"

$\exists$ stands for "there exists,"

$\ni$ stands for "such that,"

$\exists!$ stands for "there exists a unique,"

$\mathbb{N}$ is the *black board font* for the set of natural numbers, $\mathbb{N} = \{1, 2, 3, \ldots\}$,

$\mathbb{W}$ is the *black board font* for the set of whole numbers, $\mathbb{W} = \{0, 1, 2, \ldots\}$,

$\mathbb{Z}$ is the *black board font* for the set of integers, $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$,

$\mathbb{Q}$ is the *black board font* for the set of rational numbers, $\mathbb{Q} = \{p/q \mid p, q \in \mathbb{Z}, q \neq 0\}$, and

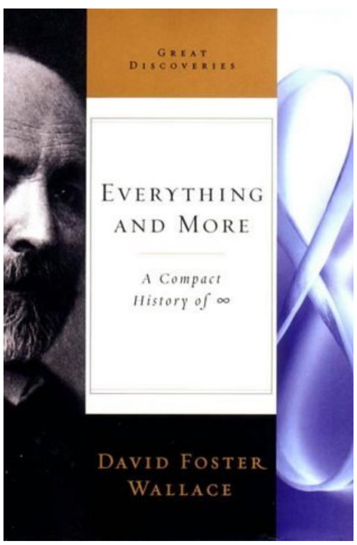
$\mathbb{R}$ is the *black board font* for the set of real numbers.

Note. In the Preface, Kirkwood states that the only prerequisite for his book is a calculus sequence, and that a previous course in abstract math is not assumed. As a consequence, the first few sections give a "gentle" transition into the abstract, rigorous setting. However, ETSU has Mathematical Reasoning (MATH 3000) as a prerequisite for this class, so we will not dwell too much on the background material (such as going through the Principle of Mathematical Induction in depth). I have online notes posted for Mathematical Reasoning, covering logic, sets and their cardinalities, and number theory.

Note. Kirkwood comments in the Introduction (see page 2): "Definitions are to be interpreted in the biconditional (if and only if) form, even though it has become customary not to state them in that fashion." He also addresses (very briefly) two properties of axiomatic systems: *consistency* and *independence* (see page 1). These topics are covered in much more depth in Introduction to Modern Geometry (MATH 4157/5157). See my online notes for that class on Section 1.4. Consistency and Section 1.5. Independence.

Note. These notes contain occasional references to the history of the topic at hand. An excellent online source for biographical information on mathematicians is the MacTutor History of Mathematics webpage. The history of analysis is on the menu of topics in History of Mathematics (MATH 3040), but there is a lot on the menu of that class, so this material may be covered (or not covered in detail). I have some online notes for History of Mathematics after 1600 in preparation which will cover this material; see Chapter 14, "The Later Nineteenth Century and the Arithmetization of Analysis."

Note. A really nice popular level history of many of the results of this class are in David Foster Wallace's *Everything and More: A Compact History of* ∞ (W.W. Norton & Company, 2003). Wallace is better known as a novelist and writer. He wrote *Infinite Jest* (1996) and his *The Pale King* (published posthumously in 2011) was a finalist for a Pulitzer Prize in 2012. *Everything and More* is laid out in a most unconventional way. It has no table of contents, the chapters do not have titles, and there is no index. It does have a bibliography and a healthy dose of footnotes that document sources throughout the book. It is part of the W.W. Norton "Great Discoveries" series (which includes books on the Pierre and Marie Curie, Einstein, Hubble, Gödel, and Alan Turing).



The first three chapters address philosophy and epistemology, the mathematics of the Pythagoreans and ancient Greeks (especially Zeno, Eudoxus, and the Method of Exhaustion; these ideas evolved into the foundations of calculus), the Arabic contributions, and Renaissance era mathematics. The fourth chapter covers Newton and Leibniz's work. Chapters 5 and 6 address the rigorous foundations of calculus during the 19th century. Key characters in this part of the story are Cauchy, Dirichlet, Weierstrass, and Dedekind. *This* is the material on which we concentrate in this class. The final chapter discusses Cantor, transfinite numbers, formalism, and the Continuum Hypothesis (topics which we will briefly explore). *Everything and More* is an excellent history of the Analysis 1 material! It includes some explanations of the material (some of it informal, but some of it surprisingly technical for a popular level book).

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