Analysis 1, Chapter 1 Study Guide

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The following is a *brief* list of topics covered in Chapter 1 of Kirkwood's *An Introduction to Analysis*, and the associated supplements. This list is not meant to be comprehensive, but gives a list of several important topics.

Chapter 1. The Real Number System.

1-1. Sets and Functions. Undefined terms ("set" and "element"), Russell's Paradox and the idea of a "biggest set" (Note 1.1.A), empty set, complement of a set, iniversal set, set builder notation, subset, superset, equal sets, proper and improper subsets, union, intersection, relative complement, indexed sets, disjoint sets, Cartesian product of sets, relative complements of unions and intersections (Theorem 1-1), DeMorgan's Laws (Corollary 1-1), functions between sets, range, image, domain, equal functions, one-to-one function, test for one-to-one (Note 1.1.E), onto function, sum/difference/product/quotient of functions, function compositions, compositions of one-to-one and of onto functions (Theorem 1-2), inverse function, domains and ranges of a function and its inverse (Note 1.1.G), identity function, inverse image, $\sqrt{9}$ (Note 1.1.H), functions and inverse functions with unions and intersections (Exercise 1.1.13.

1-2. Properties of the Real Numbers as an Ordered Field. Ring/commutative ring/ring with unity/division ring/field (Note 1.2.A), definition of field in terms of axioms (Axioms 1–7), binary operations, associative, commutative, distribution, additive identity, multiplicative identity, inverses, there's no such thing as subtraction and division (Note 1.2.B), examples of fields, constructible numbers, uniqueness of identities and inverses (Theorems 1-3 and 1-4), Axiom of Order (Axiom 8), positive set, Law of Trichotomy), examples of ordered fields, examples of fields that are not ordered, order, interaction of the order and the field operations (Theorem 1-7), interval, degenerate interval, axiomatic development of $\mathbb{N}$ and $\mathbb{Q}$ and properties of integer exponents (Note 1.2.C), the existence of nth roots (Theorem 1-8; not proved in Section 1-2), rational exponents, inequalities and exponents (Theorems 1-9 and 1-10), some properties of the ordering (Exercise 1.2.7), Principle of Mathematical Induction, binomial coefficients, summation of binomial coefficients (Theorem

1-11), The Binomial Theorem (Theorem 1-12), absolute value, absolute value in equalities and inequalities (Theorem 1-13), The Triangle Inequality for absolute value (Theorem 1-13(h)), definition of a metric, the Euclidean metric, the taxicab metric.

1-3. The Completeness Axiom. Definitions of upper bound/bounded above/lower bound/bounded below/unbounded, least upper bound, greatest lower bound, definition of complete ordered field, Axiom of Completeness (Axiom 9), there is a unique complete ordered field (Note 1.3.B), the rationals and the algebraic numbers are not complete, uniqueness of lub and glb (Theorem 1-14), definition of x^r for x positive and r rational, ε properties and lub/glb (Theorems 1-15 and 1-16), definition of the multiplication of a set of real numbers by a real number, effect of constant multiples on lub and glb (Theorem 1-17), Archimedes and his fifth axiom in *On the Sphere and Cylinder*, the Archimedean Principle (Theorem 1-18 and Corollary 1-18), Example 1.11 (proof of a lub for a given set), rational and irrational numbers between real numbers (Exercise 1.3.4), definition of same cardinality of sets, finite and infinite sets, countable and uncountable sets, union of a countable collection of countable sets is countable (Theorem 1-19), $\mathbb{Q}$ is countable (Note 1.3.D), interval $(0, 1)$ is not countable (Theorem 1-20) and the Cantor diagonalization argument, power set, cardinal number, Cantor's Theorem ($|X| < |\mathcal{P}(X)|$; Theorem 1-21), Cantor's introduction of transfinite numbers, proof of Theorem 1-8 (Exercise 1.3.9), cardinality of $A \setminus B$ (Exercise 1.3.10), biography of Georg Cantor (Note 1.3.E).