Analysis 1, Chapter 3 Study Guide

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The following is a *brief* list of topics covered in Chapter 3 of Kirkwood's *An Introduction to Analysis*, and the associated supplements. This list is not meant to be comprehensive, but gives a list of several important topics.

Chapter 3. Topology of the Real Numbers

3-1. Topology of the Real Numbers. Open set of real numbers, motivation for using open sets in analysis (Note 3.1.A), $\varnothing$ and $\mathbb{R}$ are open sets, open intervals are open sets (Theorem 3-1), closed set of real numbers, $\varnothing$ and $\mathbb{R}$ are closed sets, closed intervals (Corollary 3-1), intersections and unions of open sets (Theorem 3-2), a countable intersection of open sets may not be open (Note 3.1.C), intersections and unions of closed sets (Theorem 3-3), a countable union of closed sets may not be closed (Note 3.1.D), definition of a topology on a set, the usual topology on $\mathbb{R}$ (Example 3.1.A), the nine topologies on $X = \{a, b, c\}$, discrete topology, trivial topology, finer/courser topologies, closed topological space, open/closed relative to a set, both open and closed in connection with connectivity (Note 3.1.F), intersections and unions of sets open relative to A, classification of open sets of real numbers (Theorem 3.5), interior point of a set, boundary point of a set, limit point of a set, isolated point of a set, a point that is not in a set and not a boundary point of the set (Note 3.1.H), classification of closed sets in terms of boundary points (Theorem 3-6), properties of boundary points and limit points (Exercise 3.1.15), classification of closed sets in terms of limit points (Corollary 3-6(a)), classification of open sets in terms of boundary points (Corollary 3-6(b)), closure of a set, the closure of a set is the set along with its boundary points (Note 3.1.I), the closure of a set is closed, cover of a set, open cover of a set, finite cover of a set, compact set, the importance of compact sets (Note 3.1.J), a countable intersection of closed nested sets in nonempty (Theorem 3-8), Lindelöf Property (Theorem 3-9), Henie-Borel Theorem (Theorem 3-10; closed and bounded sets are compact), compact sets in finite and infinite dimensional spaces (Note 3.1.K), compact sets are closed and bounded (Theorem 3-11), Theorem 3-11 holds in other settings (Note 3.1.L), infinite subsets of compact sets (Theorem 3-12), classifications of compact sets of real numbers (Theorem 3-13), connected set, separation of a set (Note 3.1.M), sets that are not connected have more sets that are both open and closed relative to the set (Note 3.1.N), connected sets of real numbers are either intervals or singletons (Theorem 3-14 and Theorem 3-1-A), brief biographies of Eduard Heine and Émile Borel (Note 3.1.O), measure theory and probability theory (Note 3.1.O).

Supplement. A Classification of Open Sets of Real Numbers. Exercise 3.1.4, maximal open intervals of an open set (constructed from the results of Exercise 3.1.4), an open set is partitioned by its maximal open intervals (Exercise 3.1.5), the classification of open sets of real numbers (Theorem 3-5), other classes of sets (G_δ sets; Note OS.B), the classification of closed sets of real numbers (Theorem OS.1), complications of classifying closed sets directly (Note OS.C), other classes of sets (F_σ sets; Note OS.D), Borel sets (Note OS.E).