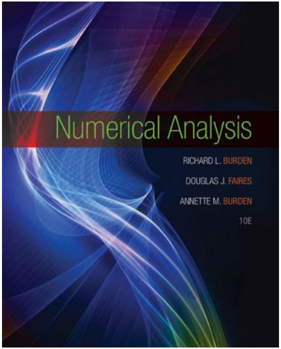
Numerical Analysis

Chapter 2. Solutions of Equations in One Variable

2.1. The Bisection Method—Proofs of Theorems



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Theorem 3.3

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$$|p_n - p| \leq \frac{b-a}{2^n} \text{ when } n \geq 1.$$

Proof. For each $n \geq 1$, we have

$$b_n - a_n = \frac{1}{2^{n-1}}(b-a) \text{ and } p \in (a_n, b_n).$$

Since $p_n = (a_n + b_n)/2$ for all $n \geq 1$, we have

$$|p_n - p| \leq \frac{1}{2}(b_n - a_n) = \frac{1}{2}\frac{b-a}{2^{n-1}} = \frac{b-a}{2^n},$$

as claimed. □

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