Preface

Note. With $\pi(x)$ and the number of prime numbers less than or equal to x, the Prime Number Theorem states that $\pi(x) \sim x/\log x$. The result lies in the area of analytic number theory and the first proofs of it (in the 1896) require hardly any number theory and a lot of analysis (namely, complex analysis). Jameson says (page vii): "For this reason, it does not fit very comfortably in books on analytic number theory, often appearing as an outlying topic. ...It is important and distinctive enough to be treated as a subject in its own right, rather than a fringe topic of either number theory or analysis." That is the motivation for this book (though some core analytic number theory results).

Note. Jameson's book is written at the standard senior-undergraduate/first-year-graduate-student level. The proclaimed prerequisite material is "standard undergraduate courses in both real and complex analysis," including integration and uniform convergence. This is the material covered in ETSU's Analysis 1 (MATH 4127/5127), Analysis 2 (MATH 4137/5137), and Complex Variables (MATH 4337/5337). Some exposure to graduate level Complex Analysis 1 (MATH 5510) could be helpful, but measure theory (which is covered in Real Analysis 1 [MATH 5210]) is avoided. New ideas encountered in this book include the Euler product representation of the zeta function, Dirichlet series and their convolutions and inversions, Möbius inversion, and extensions of the zeta function in ways not involving all the background of analytic continuation.

Note. The basic coverage of the Prime Number Theorem is given in the first three chapters (Chapter 1. Foundations, Chapter 2. Some Important Dirichlet Series and Arithmetic Functions, and Chapter 3. The Basic Theorems). Chapter 3 gives two proofs of the Prime Number Theorem. One is a variant of the method involving inversion of Dirichlet series (due to Hjalmar Mellin's "Mellin Transform" and Oskar Perron's "Perron's formula"), and the other is Newman's methods "which has attracted considerable attention since its appearance in 1980." (See page ix). D. J. Newman's paper is "Simple Analytic Proof of the Prime Number Theorem," *The American Mathematical Monthly*, **87**(9), 693–696 (1980); this is available on JSTOR (which will require your ETSU username and password to access).

Note. Chapter 4, "Prime Numbers in Residue Classes: Dirichlet's Theorem," concerns Dirichlet's Theorem on the equal distribution of prime numbers among residue classes (this chapter requires a bit of knowledge of group theory). Chapter 5, "Error Estimates and the Riemann Hypothesis," deals with error estimates in the approximation of $\pi(x)$ by $x/\log x$ (and other approximations). This is connected with location of zeros of the Riemann zeta function and the Riemann Hypothesis. Chapter 6, "An 'Elementary' Proof of the Prime Number Theorem," covers the proof from 1949 by Selberg and Erdős which is 'elementary' in the sense that it does *not* use complex analysis, but only traditional number theoretic arguments. Notice that the text also includes exercises, making this a suitable text book on the Prime Number Theorem.

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