Theory of Matrices, MATH 5090, Summer 2018

Homework 4, Section 3.1

Due Tuesday, June 19 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

3.1(a) Give a basis for $\mathbb{R}^{n\times m}$ for $n \geq m$. Verify that your set satisfies the definition of basis.

3.1(c) Give a basis for the vector space of all $n \times n$ symmetric matrices. Verify that your set satisfies the definition of basis.

3.1.A. (Corollary 3.1.D.) If a row or column of $n \times n$ matrix A is a scalar multiple of another row or column (respectively) of A, then $\det(A) = 0$.

3.1.B. Consider the $n \times n$ matrix $A = \begin{bmatrix} x+\lambda & x & \cdots & x \\ x & x+\lambda & \cdots & x \\ \vdots & \vdots & \ddots & \vdots \\ x & x & \cdots & x+\lambda \end{bmatrix}$. Use Theorem 3.1.E to show that $\det(A) = \lambda^{n-1}(nx+\lambda)$. HINT: Add the last $n-1$ columns of A to the first column, and then subtract the first row of the resultant matrix from each of the last $n-1$ rows.