Theory of Matrices, MATH 5090, Summer 2018

Homework 8, Sections 3.4, 3.5, 3.7

Due Tuesday, July 3 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

3.13. Prove Theorem 3.4.1: If A is a square nonsingular matrix and $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ where both A_{11} and A_{22} are nonsingular then in terms of the Schur complement of A_{11} in A, $Z = A_{22} - A_{21}A_{11}^{-1}A_{12}$, we have the inverse of A is

$$A^{-1} = \begin{bmatrix} A_{11}^{-1} + A_{11}^{-1}A_{12}Z^{-1}A_{21}A_{11}^{-1} & -A_{11}^{-1}A_{12}Z^{-1} \\ -Z^{-1}A_{21}A_{11}^{-1} & Z^{-1} \end{bmatrix}.$$

3.8. Prove that a square matrix that is either row or column diagonally dominant is nonsingular.

3.9. Prove that a positive definite matrix is nonsingular.

3.7.A. Prove Theorem 3.7.4: If A and B are orthogonal then the Kronecker product $A \otimes B$ is orthogonal. HINT: Use Theorem 3.2.5 and show $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$.