Theory of Matrices, MATH 5090, Summer 2020

Homework 1, Section 2.1

Due Thursday, June 11

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

2.3 Let $\{v_i\}_{i=1}^n$ be an orthonormal basis for the n-dimensional vector space V. Let $x \in V$ have the representation $x = \sum_{i=1}^n b_i v_i$ for scalars b_i. Prove that the Fourier coefficients b_i satisfy $b_i = \langle x, v_i \rangle$.

2.6. Prove the following inequalities.

(a) Prove Young's Inequality: For any p and q such that $p > 1$, $q > 1$, and $\frac{1}{p} + \frac{1}{q} = 1$ and for any positive a and b,

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}.$$

(b) Prove Hölder's Inequality: For any p and q such that $p > 1$, $q > 1$, and $\frac{1}{p} + \frac{1}{q} = 1$ and for vectors x and y in $\mathbb{R}^n$, $|\langle x, y \rangle| \leq \|x\|_p \|y\|_q$.

(c) Prove the Triangle Inequality for any ℓ^p norm. This is sometimes called Minkowski's Inequality.

2.1.A. Prove Theorem 2.1.1(S1): Let $x, y \in \mathbb{R}^n$ and let $a \in \mathbb{R}$ be a scalar. Then:
$a(x + y) = ax + ay$ (Distribution of Scalar Multiplication over Vector Addition).