Theory of Matrices, MATH 5090, Summer 2020

Homework 2, Sections 2.1 and 2.2, Solutions

Due Tuesday, June 16

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

2.1.D. Prove Theorem 2.1.9: Equivalence of norms is an equivalence relation. That is, it is reflexive, symmetric, and transitive. (Section 2.1.5)

2.9. Prove that the intersection of two orthogonal vector spaces consists only of the zero vector. (Section 2.1.8)

2.11. In $\mathbb{R}^2$, the diagonally directed line segment with direction vector $[1, 1]$ makes an angle of $45°$ with each of the positive axes since

$$\frac{\langle [1,1],[1,0]\rangle}{\|[1,1]\|\,\|[1,0]\|} = \frac{\langle [1,1],[0,1]\rangle}{\|[1,1]\|\,\|[0,1]\|} = \frac{1}{\sqrt{2}}$$

and $\cos^{-1}(1/\sqrt{2}) = 45°$. (a) In 3 dimensions, what is the angle between $[1, 1, 1]$ and each of the positive axes? (b) In 10 dimensions? (c) In 100 dimensions? (d) in 1000 dimensions? Gentle comments: "We see that in higher dimensions any two lines are almost orthogonal.

2.13(c) For $x, y, z \in \mathbb{R}^3$, prove that $\langle x, y \times z\rangle = \langle x \times y, z\rangle$. NOTE: This is called the scalar triple product.

2.13(d) Let $x, y, z \in \mathbb{R}^3$. Prove that $x \times (y \times z) = \langle x, z\rangle y - \langle x, y\rangle z$. NOTE: This is called the "triple vector product."