Theory of Matrices, MATH 5090, Summer 2020

Homework 4, Sections 3.1 and 3.2

Due Tuesday, June 23

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

3.1.A. (Corollary 3.1.D.) If a row or column of $n \times n$ matrix A is a scalar multiple of another row or column (respectively) of A, then $\det(A) = 0$.

3.1.B. Consider the $n \times n$ matrix $A = \begin{bmatrix} x+\lambda & x & \cdots & x \\ x & x+\lambda & \cdots & x \\ \vdots & \vdots & \ddots & \vdots \\ x & x & \cdots & x+\lambda \end{bmatrix}$. Use Theorem 3.1.E to show that $\det(A) = \lambda^{n-1}(nx+\lambda)$. HINT: Add the last $n-1$ columns of A to the first column, and then subtract the first row of the resultant matrix from each of the last $n-1$ rows.

3.3. Prove that for any quadratic form $x^T A x$ there is a symmetric matrix A_s such that $x^T A_s x = x^T A x$. HINT: Let $A_s = \frac{1}{2}(A + A^T)$.

3.2.A. Prove Theorem 3.2.1 part (4): Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be $n \times n$ matrices. If A and B are diagonal then AB is diagonal. If A and B are upper triangular then AB is upper triangular. If A and B are upper triangular then AB is upper triangular. If A and B are lower triangular then AB is lower triangular.