

Theory of Matrices, MATH 5090, Summer 2020

Homework 7, Sections 3.5, 3.6, and 3.7

Due Tuesday, July 7

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

3.8. Prove that a square matrix that is either row or column diagonally dominant is nonsingular.

3.9. Prove that a positive definite matrix is nonsingular.

3.15. Confirm that $B = R^T(L^T A R^T)^{-1} L^T$ in the proof of Theorem 3.6.2 satisfies the four parts of the definition of pseudoinverse of A (so $A^+ = B$). HINT: Show that $B = R^T(RR^T)^{-1}(L^T L)^{-1} L^T$ and use this in the computations.

(1) $ABA = A$,

(2) $BAB = B$,

(3) BA is symmetric, and

(4) AB is symmetric.

3.7.A. Prove Theorem 3.7.4: If A and B are square orthogonal then the Kronecker product $A \otimes B$ is orthogonal. HINT: Use Theorem 3.2.5 and show $(A \otimes B)^{-1} = (A \otimes B)^T$.