Theory of Matrices, MATH 5090, Summer 2020

Homework 8, Section 3.8

Due Friday, July 10

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

3.16. (d) Prove Theorem 3.8.2:

Basic Properties of Eigenvalues and Eigenvectors.

Let A be an $n \times n$ real matrix with eigenpair c and v.

(4) If A is square and $c \neq 0$ then $1/c$ and v are an eigenpair for A^+, where A^+ is the Moore-Penrose inverse of A (or pseudoinverse of A).

3.16. (e) Prove Theorem 3.8.2(5):

Basic Properties of Eigenvalues and Eigenvectors.

Let A be an $n \times n$ real matrix with eigenpair c and v.

(5) If A is diagonal or triangular with diagonal entries a_{ii}, then the eigenvalues of A are a_{ii}. For A diagonal, the corresponding eigenvectors are e_i (the ith unit vector in $\mathbb{R}^n$).

3.18. Prove Theorem 3.8.7(5): Let A be a real square matrix and (c, v) an eigenpair (possibly complex) for A. Prove that c is real if A is symmetric.

3.8.B. Let A be an $n \times n$ (not necessarily symmetric) matrix. Let w be a left eigenvector for eigenvalue c and let v be a right eigenvector for eigenvalue c, where $w^T v = 1$. Prove that for $k \in \mathbb{N}$, $(A - cvw^T)^k = A^k - c^k v w^T$.