This contradiction shows that the assumption that \( C \) is measurable is false.

\[
0 = (\gamma + 2 \varepsilon) w(\varepsilon) = \lambda^{(0) w(\varepsilon)}(\gamma + \varepsilon) w(\varepsilon) > 0
\]

Monotonicity (Lemma 2.14): Suppose \( w(\varepsilon) > 0 \). So, we have a CONTRADICTION by

the property of \( \varepsilon \text{-} \lambda \text{-} \mu \text{-} \beta \). Since \( x \) is arbitrary,

\( x = \gamma + \varepsilon \). Then there is some \( \varepsilon \text{-} \mu \text{-} \beta \text{-} \gamma \) such that \( x = \gamma + \varepsilon \text{ and } c = \beta \).

Intuitively, let \( x \in \varepsilon \text{-} \mu \text{-} \beta \). Then \( \mu \text{-} \beta \text{ is bounded and countably}

Proof (continued). Since \( \mu \) is bounded, then \( \varepsilon \text{-} \gamma \text{-} \beta \text{-} \gamma \) for some

Theorem 2.17. The Vitali Construction of a Nonmeasurable Set

\[
\forall \gamma \exists \mu (\varepsilon) \subseteq \gamma \text{ and so } w(\varepsilon) = 0 \text{ otherwise.}
\]

Then \( \forall \gamma \exists \mu (\varepsilon) \subseteq \gamma \) and so \( w(\varepsilon) = 0 \text{ otherwise.}

Proof. The translate of a measurable set is measurable by Proposition 2.13.

which is the collection of translates of \( \varepsilon \text{-} \gamma \text{-} \beta \text{-} \gamma \) is disjoint. Then

Suppose there is a bounded countably infinite set of real numbers \( A \) for

Lemma 2.16. Let \( \varepsilon \in \gamma \) be a bounded measurable set of real numbers

Real Analysis

Chapter 2. Lebesgue Measure

2.6. Nonmeasurable Sets (4th Ed.)—Proofs of Theorems
Theorem 2.18. There are disjoint sets of real numbers $A$ and $B$ for which

\[ (g) \quad m^* A \neq m^* B. \]

Proof. Assume $m^* (A \cap B) = 0$ for every disjoint pair of

\[ (g) \quad m^* A + m^* B > m^* (A \cup B). \]

So for some disjoint $A, B \subseteq \mathbb{R}$, we have $m^* (A \cap B) = 0$. By

subadditivity (Proposition 2.3), $m^* A \cup m^* B \geq m^* (A \cup B)$, so it must be

that for some disjoint $A, B \subseteq \mathbb{R}$, we have $m^* (A \cap B) > 0$.