false and so \( f \neq 0 \) is not measurable.

\[ a \leq f \leq b \] for all \( x \in \mathbb{R} \).

Therefore the assumption that \( f \) is measurable is

\[ \{0\} \cap \mathbb{N} = \{0\} \]

since \( \mathbb{N} \) is a countable set.

By the proposition of additivity, \( \mathbb{N} \) is the union of two countable sets.

Therefore, \( \mathbb{N} \) is measurable.

Hence \( \mathbb{N} \) is measurable.

\[ \{0\} \cap \mathbb{N} = \{0\} \]

and so \( \mathbb{N} \) is measurable.

\[ \mathbb{N} \]

Theorem 2.6.B (continued)

Proof.

First, we establish some set theoretic results. Let \( \{1, 2, 3\} \) be an enumeration of \( \mathbb{N} \).

For every \( \mathbb{N} \) in \( \{1, 2, 3\} \), define \( \mathbb{N} \) as follows:

\[ \mathbb{N} \]

Lemma 2.6.A.

Set \( f \) is not measurable.

Lemma 2.6.A.
\( A, B \subseteq \mathbb{R} \) we have \( m^* (A \cup B) = m^* (A) + m^* (B) \), so it must be that for some disjoint

\( m^* (A \cap B) \neq m^* (A) + m^* (B) \). By subadditivity (Proposition 2.3),

so for some disjoint \( A, B \subseteq \mathbb{R} \) we have

and so every \( E \subseteq \mathbb{R} \) is measurable. A CONTRADICATION to Corollary

\( \forall \epsilon \exists \delta \forall E \subseteq \mathbb{R} \) we have

sets \( A \) and \( B \). Then for any \( A, E \subseteq \mathbb{R} \) we have

Proof. Assume \( m^* (A \cup B) = m^* (A) + m^* (B) \) for every disjoint pair of

There are disjoint sets of real numbers \( A \) and \( B \) for which

Theorem 2.18