Real Analysis 1, MATH 5210

Homework 1, Section 1.4

Due Wednesday September 10, 2014 at 4:00

Prove each of the following.

Proposition 13. Let $\mathcal{C}$ be a collection of subsets of a set X. Then the intersection $\mathcal{A}$ of all σ-algebras of subsets of X that contain $\mathcal{C}$ is a σ-algebra and it is the smallest σ-algebra containing $\mathcal{C}$.

1.35. The collection of Borel sets is the smallest σ-algebra that contains the closed sets.

1.58(c). Let f be a continuous real-valued function on $\mathbb{R}$. The inverse image of a Borel set is Borel. HINT: For any function, $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$, and $f^{-1}(\mathbb{R} \setminus B) = \mathbb{R} \setminus f^{-1}(B)$. Consider $\mathcal{E}$ the collection of sets such that $E \in \mathcal{E}$ implies that $f^{-1}(E)$ is Borel. Show that $\mathcal{E}$ is a σ-algebra containing all open sets (so $\mathcal{E} \supset \mathcal{B}$).

1.56. (Bonus) Let f be a real-valued function defined on all of $\mathbb{R}$. The set of points at which f is continuous is a G_δ set.