Real Analysis 1, MATH 5210

Homework 10, Section 4.4, Corrected

Due Wednesday November 26, 2014 at noon

Prove each of the following.

Problem 4.30. Let g be a nonnegative integrable function over E and suppose $\{f_n\}$ is a sequence of measurable functions on E (not necessarily nonnegative) such that for each $n \in \mathbb{N}$, $|f_n| \le g$ a.e. on E. Then

$$\int_E (\liminf f_n) \le \liminf \left(\int_E f_n \right) \le \limsup \left(\int_E f_n \right) \le \int_E (\limsup f_n).$$

HINT: Apply the Generalized Fatou's Lemma (problem 4.27) to sequences $\{f_n + g\}$ and $\{g - f_n\}$.

Problem 4.32. Prove the General Lebesgue Dominated Convergence Theorem by following the proof of the Lebesgue Dominated Convergence Theorem, but replacing the sequences $\{g - f_n\}$ and $\{g + f_n\}$, respectively, by $\{g_n - f_n\}$ and $\{g_n + f_n\}$.

Problem 4.33. Let $\{f_n\}$ be a sequence of integrable functions on E for which $f_n \to f$ a.e. on E and f is integrable over E. Then $\int_E |f - f_n| \to 0$ if and only if $\lim \int_E |f_n| = \int_E |f|$. **Hint:** (1) Assume $\int_E |f_n - f| \to 0$. Use the Integral Comparison Test. (2) Assume $\int_E |f_n| \to \int_E |f|$. Apply Fatou's Lemma to $|f_n| + |f| - |f_n - f|$.

Problem 4.34. Let f be a nonnegative measurable function on $\mathbb{R}$. Prove that $\lim_{n\to\infty} \left(\int_{-n}^{n} f \right) = \int_{\mathbb{R}} f$. **Hint:** Use Exercise 4.28.