Real Analysis 1, MATH 5210

Homework 2, Sections 2.1 and 2.2

Due Friday September 19, 2014 at 1:30

Prove each of the following.

2.2. Let m' be a set function defined for all sets in a σ-algebra $\mathcal{A}$ with values in $[0, \infty]$. Assume m' is countably additive over countable disjoint collections of sets in $\mathcal{A}$. Prove that if there is a set A in the collection $\mathcal{A}$ for which $m'(A) < \infty$, then $m'(\varnothing) = 0$.

2.3. Let m' be a set function defined for all sets in a σ-algebra $\mathcal{A}$ with values in $[0, \infty]$. Assume m' is countably additive over countable disjoint collections of sets in $\mathcal{A}$. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of sets in $\mathcal{A}$. Prove that $m'(\cup_{k=1}^{\infty} E_k) \leq \sum_{k=1}^{\infty} m'(E_k)$.

2.6. Let A be the set of irrational numbers in the interval $[0, 1]$. Prove that $m^*(A) = 1$.

2.7. Prove that for any bounded set E, there is a G_δ set G for which $E \subseteq G$ and $m^*(G) = m^*(E)$. Set G is called the *outer approximation* in Section 2.4. (In fact, this result also holds if set E is not bounded.)

2.9. Prove that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.