Real Analysis 1, MATH 5210

Homework 3, Sections 2.3 and 2.4

Due Friday September 26, 2014 at 1:30

Prove each of the following.

2.14. Prove that if a set E has finite positive outer measure, then there is a bounded subset of E that also has positive outer measure. HINT: Consider the contrapositive.

2.15. Prove that if E has finite measure and $\varepsilon > 0$, then E is the disjoint union of a finite number of measurable sets, each of which has measure at most ε. HINT: Cut set E into bounded pieces and a single unbounded piece which is small in measure.

2.17. Prove that a set E is measurable if and only if for each $\varepsilon > 0$, there is a closed set F and open set $\mathcal{O}$ for which $F \subseteq E \subseteq \mathcal{O}$ and $m^*(\mathcal{O} \setminus F) < \varepsilon$. HINT: Use Theorem 2.11.

2.18. (REVISED from the text's version.) Let $m^*(E) < \infty$. Then if there exists F_σ set F and G_δ set G with $F \subseteq E \subseteq G$ and $m * (F) = m^*(E) = m^*(G)$, then E is measurable. NOTE: The text's statement is incorrect since it does not assume that E is measurable. We know (from page 3 of the class notes for Section 2.3) that there exist F_σ set F and G_δ set G, called the inner approximation and outer approximation, such that $\lambda_*(F) = \lambda_*(E) \leq \lambda^*(E) = \lambda^*(G)$ (in terms of inner measure λ_* and outer measure λ^*). So the text's conclusion holds only if set E is measurable (which it did not assume). HINT: You may assume this behavior of inner and outer measure. Use the definition of Lebesgue measurable in terms of inner and outer measure.