Real Analysis 1, MATH 5210

Homework 4, Sections 2.4 and 2.5

Due Friday October 3, 2014 at 1:30

Prove each of the following.

2.19. Let E have finite outer measure. Prove that if E is <u>not</u> measurable, then there is an open set $\mathcal{O}$ containing E that has finite outer measure and for which

$$m^*(\mathcal{O} \setminus E) > m^*(\mathcal{O}) - m^*(E).$$

NOTE: This is our first encounter with the behavior of a (Lebesgue) non-measurable set. It will get weirder.

2.24. Prove that if E_1 and E_2 are measurable, then

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$

NOTE: This is the kind of behavior we expect! Notice that, if all each set has finite measure, this reduces to the familiar $m(E_1 \cup E_2) = m(E_1) + m(E_2) - m(E_1 \cap E_2)$ which should remind you of the behavior of the *probability* P of *events* E_1 and E_2: $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

2.26. Let $\{E_k\}_{k=1}^{\infty}$ be a countable disjoint collection of measurable sets. Prove that for any set $A \subseteq \mathbb{R}$, we have

$$m^*\left(A \cap \bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} m^*(A \cap E_k).$$