Real Analysis 1, MATH 5210

Homework 5, Section 2.6

Due Friday October 10, 2014 at 1:30

Prove each of the following.

Problem A. Show that if $E \in \mathcal{M}$ and $E \subset P$, then $m(E) = 0$. HINT: Let $E_i = E \mathring{+} r_i$, where $\mathbb{Q} = \{r_i\}_{i=1}^{\infty}$. Then $\{E_i\}_{i=1}^{\infty}$ is a disjoint sequence of measurable sets and $m(E_i) = m(E)$. Therefore $\sum m(E_i) = m(\cup E_i) \leq m([0,1))$.

Problem B. Show that if A is any set with $m^*(A) > 0$, then there is a nonmeasurable set $E \subset A$. HINT: If $A \subset (0,1)$, let $E_i = A \cap P_i$. The measurability of E_i implies $m(E_i) = 0$, while $\sum m^*(E_i) \geq m^*(A) > 0$.

Problem C. Give an example $\{E_i\}_{i=1}^{\infty}$ of a disjoint sequence of sets and $m^*(\cup E_i) < \sum m^*(E_i)$. Use set P and explain.