Real Analysis 1, MATH 5210

Homework 6, Section 3.1, Solutions

Due Friday October 24, 2014 at 1:30

Prove each of the following.

Problem 3.3. Suppose a function f has a measurable domain E and is continuous except at a finite number of points. Is f necessarily measurable? If so, then prove it. If not, then give an example.

Problem 3.4. Suppose f is a real-valued function defined on $\mathbb{R}$ such that $f^{-1}(\{c\}) \in \mathcal{M}$ for each number c. Is f necessarily measurable? If so, then prove it. If not, then give an example.

Problem 3.5. Suppose the function f is defined on a measurable set E and suppose f has the property that $\{x \in E \mid f(x) > c\} \in \mathcal{M}$ for each rational number c. Is f necessarily measurable? If so, then prove it. If not, then give an example.

Problem 3.7. Bonus. Let the function f be defined on a measurable set E. Prove that f is measurable if and only if for each Borel set A, $f^{-1}(A)$ is measurable.