Real Analysis 1, MATH 5210

Homework 7, Section 3.2

Due Friday October 31, 2014 at 1:30

Prove each of the following.

Problem 3.12. Let f be a bounded measurable function on E. Prove there are sequences of simple functions on E, $\{\varphi_n\}$ and $\{\psi_n\}$, such that $\{\varphi_n\}$ is increasing and $\{\psi_n\}$ is decreasing and each of these sequences converges to f uniformly. HINT: Use partitions and refinements of these partitions.

Problem 3.14. Let f be a measurable function on E that is finite a.e. on E and $m(E) < \infty$. Prove that for each $\varepsilon > 0$, there is a measurable set F contained in E such that f is bounded on F and $m(E \setminus F) < \varepsilon$.

Problem 3.15. Let f be a measurable function on E that is finite a.e. on E and $m(E) < \infty$. Prove that for each $\varepsilon > 0$ there is a measurable set F contained in E and a sequence $\{\varphi_n\}$ of simple functions on E such that $\{\varphi_n\} \to f$ uniformly on F and $m(E \setminus F) < \varepsilon$. HINT: Use Exercises 3.12 and 3.14.