Real Analysis 1, MATH 5210, Fall 2016

Homework 10, Lebesgue Integral of Measurable Nonnegative Functions

Due Friday, November 18, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

4.17. Let E be a set of measure zero and define $f = \infty$ on E. Prove that $\int_E f = 0$.

4.20. Let $\{f_n\}$ be a sequence of nonnegative measurable functions that converges to f pointwise on E. Let $M \geq 0$ be such that $\int_E f_n \leq M$ for all $n \in \mathbb{N}$.

(a) Prove that $\int_E f \leq M$.

(b) Prove that the result in part (a) is equivalent to Fatou's Lemma. HINT: Let $M = \liminf\{\int_E f_n\}$ and let $\varepsilon > 0$. Show that $\int_E f_{n_k} < M + \varepsilon$ for some subsequence $\{\int_E f_{n_k}\}$ of $\{\int_E f_n\}$.

4.23. Let $\{a_n\}$ be a sequence of nonnegative real numbers. Define the function f on $E = [1, \infty)$ by setting $f(x) = a_n$ if $n \leq x < n + 1$ for $n \in \mathbb{N}$. Prove that $\int_E f = \sum_{n=1}^{\infty} a_n$.

4.26. Show that the Monotone Convergence Theorem may not hold for decreasing sequences of functions.