Real Analysis 1, MATH 5210, Fall 2016

Homework 4, Intro to Measure

Due Friday, September 16, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

2.1.A. Let m' be a set function defined for all sets in a σ-algebra $\mathcal{A}$ with values in $[0, \infty]$. Assume m' is countably additive over countable disjoint collections of sets in $\mathcal{A}$. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of sets in $\mathcal{A}$ where $E_k \subset E_{k+1}$ for all $k \in \mathbb{N}$. Prove that $m'(\cup_{k=1}^{\infty} E_k) = \lim_{k\to\infty} m'(E_k)$. This is called *continuity with respect to increasing sequences.*

2.1.B. Let m' be a set function defined for all sets in a σ-algebra $\mathcal{A}$ with values in $[0, \infty]$. Assume m' is countably additive over countable disjoint collections of sets in $\mathcal{A}$. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of sets in $\mathcal{A}$ where $E_k \supset E_{k+1}$ for all $k \in \mathbb{N}$. Suppose $m'(E_1) < \infty$. Prove that $m'(\cap_{k=1}^{\infty} E_k) = \lim_{k\to\infty} m'(E_k)$. This is called *continuity with respect to decreasing sequences.*

2.4. A set function c, defined on all subsets of $\mathbb{R}$, is defined as follows. Define $c(E)$ to be ∞ if E has infinitely many members and $c(E)$ to be equal to the number of elements in E if E is finite; define $c(\varnothing) = 0$. Prove that c is a countably additive and translation invariant set function. This set function is called the *counting measure.* HINT: For countable additivity, consider two cases: (1) $\dot\cup_{k=1}^{N} E_k$ and $\dot\cup_{k=1}^{\infty} E_k$. For translation invariant, consider two cases: E finite and E infinite.