Real Analysis 1, MATH 5210, Fall 2016

Homework 7, Outer and Inner Approximation

Due Friday, October 14, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

2.16(a). Complete the proof of Theorem 2.11 by showing (iii) set E is measurable if and only if for all $\varepsilon > 0$ there is a closed set F with $F \subset E$ for which $m^*(E \setminus F) < \varepsilon$. HINT: E is measurable if and only if E^c is measurable, and $E \setminus \mathcal{O}^c = \mathcal{O} \setminus E^c$. To show that E is measurable given such closed set F, consider the (easily verified) fact that $A \cap E = (A \cap F) \cup (A \cap (E \setminus F))$.

2.17. Prove that a set E is measurable if and only if for each $\varepsilon > 0$, there is a closed set F and open set $\mathcal{O}$ for which $F \subseteq E \subseteq \mathcal{O}$ and $m^*(\mathcal{O} \setminus F) < \varepsilon$. HINT: Use Theorem 2.11.

2.19. Let E have finite outer measure. Prove that if E is <u>not</u> measurable, then there is an open set $\mathcal{O}$ containing E that has finite outer measure and for which

$$m^*(\mathcal{O} \setminus E) > m^*(\mathcal{O}) - m^*(E).$$

HINT: Consider the contrapositive. NOTE: This is our first encounter with the behavior of a (Lebesgue) non-measurable set. It will get weirder.