

# Real Analysis 1, MATH 5210, Fall 2016

## Homework 8, Nonmeasurable Sets & Sums Products, and

### Compositions; Solutions

Due Friday, October 28, at 1:30

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

- 2.33.** Let  $E$  be a nonmeasurable set of finite outer measure. Prove that there is a  $G_\delta$  set  $G$  that contains  $E$  for which  $m^*(E) = m^*(G)$ , while  $m^*(G \setminus E) > 0$ .
- 3.3.** Suppose a function  $f$  has a measurable domain  $E$  and is continuous except at a finite number of points. Is  $f$  necessarily measurable? If so, then prove it. If not, then give an example.
- 3.7.** Let the function  $f$  be defined on a measurable set  $E$ . Prove that  $f$  is measurable if and only if for each Borel set  $A$ ,  $f^{-1}(A)$  is measurable.