Real Analysis 1, MATH 5210, Fall 2018

Homework 9, Sequential Pointwise Limits and Simple Approximation

Due Friday, November 2, at 1:40

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses.

3.12. Let f be a bounded measurable function on E. Prove there are sequences of simple functions on E, $\{\varphi_n\}$ and $\{\psi_n\}$, such that $\{\varphi_n\}$ is increasing and $\{\psi_n\}$ is decreasing and each of these sequences converges to f uniformly. HINT: Use partitions and refinements of these partitions.

3.13. A real-valued function is said to be *semisimple* provided it takes on only a countable number of values. Let f be any measurable function. Prove that there is a sequence of semisimple functions $\{f_n\}$ on E that converges to f uniformly on E.

3.20. Let A and B be any sets. Prove that $\chi_{A\cap B} = \chi_A \cdot \chi_B$, $\chi_{A\cup B} = \chi_A + \chi_B - \chi_A \cdot \chi_b$, and $\chi_{A^c} = 1 - \chi_A$. HINT: Let U be the universal set and justify each claim for all $x \in U$.

3.7. **(Bonus)** Let the function f be defined on a measurable set E. Prove that f is measurable if and only if for each Borel set A, $f^{-1}(A)$ is measurable.