Real Analysis 1, MATH 5210, Fall 2020

Homework 1, Essential Background

Due Friday, August 28, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!** These problems should be very easy for you and based on the material of your senior-level analysis class.

0.1. Use the Axioms of Definition 0.1 to prove that the additive identity and multiplicative identity in a field are unique.

0.2. Let $\mathbb{F}$ be a field. Prove that $(-a)(-b) = ab$ for all $a, b \in \mathbb{F}$. You may assume that additive inverses are unique and $0a = 0$ for all $a \in \mathbb{F}$.

0.3. Let $A \subset \mathbb{R}$ with $\text{lub}(A) = \alpha$. Give a proof by contradiction that for all $\varepsilon > 0$, the interval $(\alpha - \varepsilon, \alpha]$ contains an element of set A.

0.4 Let $\{U_\alpha\}_{\alpha \in A}$ be a collection of disjoint open sets of real numbers where index α is from some indexing set A. Prove that $\{U_\alpha\}$ is in fact a countable collection (that is, prove that index set A is countable). You may assume that every open interval contains a rational number, the rational numbers are countable, and a set which is countably infinite is of the smallest infinite cardinality. HINT: Associate each set with a different rational number.