Real Analysis 1, MATH 5210, Fall 2020

Homework 10, 3.2. Sequential Pointwise Limits and Simple Approximation, 4.2. The Lebesgue Integral of a Bounded Measurable Function over a Set of Finite Measure

Due Wednesday, November 18, by noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

3.13. A real-valued function is said to be *semisimple* provided it takes on only a countable number of values. Let f be any measurable function. Prove that there is a sequence of semisimple functions $\{f_n\}$ on E that converges to f uniformly on E.

3.17. Let I be a closed bounded interval and ψ a simple function defined on I. Let $\varepsilon > 0$. Prove that there is a step function h on I and a measurable subset F of I for which

$$h = \psi \text{ on } F \text{ and } m(I \setminus F) < \varepsilon.$$

HINT: Use Problem 3.16. You may assume that a linear combination of step functions is a step function. NOTE: This result shows that a simple function on a bounded measurable set is "nearly" a step function.

4.9. Let E have measure zero. If f is bounded on E then f is measurable and $\int_E f = 0$.

4.11. Prove by example that the Bounded Convergence Theorem does not hold for Riemann integrals. HINT: Create sequence $\{f_n\}_{n=1}^{\infty}$ of Riemann integrable functions such that $f = \lim_{n\to\infty} f_n$ is not Riemann integrable.