Real Analysis 1, MATH 5210, Fall 2020

Homework 11, 4.2. The Lebesgue Integral of a Bounded Measurable Function over a Set of Finite Measure, 4,3. The Lebesgue Integral of a Measurable Nonnegative Function

Due Friday, December 4, by noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

4.16. Let f be a nonnegative bounded measurable function on a set E of finite measure. If $\int_E f = 0$ then $f = 0$ a.e. on E.

4.18. Prove that the integral of a bounded measurable function of finite support is properly defined. That is, it is independent of the choice of a set E_0 of finite measure. HINT: Let E_1 and E_2 be measurable subsets of E such that $m(E_1) < \infty$, $m(E_2) < \infty$, $f \equiv 0$ on $E \setminus E_1$, and $f \equiv 0$ on $E \setminus E_2$. Notice that $E_1 = (E_1 \setminus E_2) \cup (E_1 \cap E_2)$ and $E_2 = (E_2 \setminus E_1) \cup (E_1 \cap E_2)$.

4.25. Let $\{f_n\}$ be a sequence of nonnegative measurable functions on E that converges pointwise on E to f. Suppose $f_n \le f$ on E for each $n \in \mathbb{N}$. Prove that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

4.27. Prove the Generalized Fatou's Lemma: If $\{f_n\}$ is a sequence of nonnegative measurable functions in E, then

$$\int_E (\liminf f_n) \le \liminf \left(\int_E f_n \right).$$