Real Analysis 1, MATH 5210, Fall 2020

Homework 3, 1.4 Borel Sets, 2.1 Intro to Measure

Due Sunday, September 13, by noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

Proposition 13. Let $\mathcal{C}$ be a collection of subsets of a set X. Then the intersection $\mathcal{A}$ of all σ-algebras of subsets of X that contain $\mathcal{C}$ is a σ-algebra and it is the smallest σ-algebra containing $\mathcal{C}$.

1.36. The collection of Borel sets is the smallest σ-algebra that contains all intervals of the form $[a, b)$ where $a < b$.

1.56. Let f be a real-valued function defined on $\mathbb{R}$. Prove that the set of points at which f is continuous is a G_δ set. HINT: In the Riemann-Lebesgue Theorem supplement, modify the proof of Exercise 6.1.8 to show that when $\mathcal{D}(f) = \mathbb{R}$, $A_s = \{x \in \mathbb{R} \mid \text{osc}(f : x) \geq s\}$ is closed. Then use Theorem 6-10.

2.3. Let m' be a set function defined for all sets in a σ-algebra $\mathcal{A}$ with values in $[0, \infty]$. Assume m' is countably additive over countable disjoint collections of sets in $\mathcal{A}$. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of sets in $\mathcal{A}$. Prove that $m'(\cup_{k=1}^{\infty} E_k) \leq \sum_{k=1}^{\infty} m'(E_k)$.