Real Analysis 1, MATH 5210, Fall 2020

Homework 4, 2.2. Lebesgue Outer Measure Due Sunday, September 20, by noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; do your own work!!!

- **2.6.** Let A be the set of irrational numbers in the interval [0,1]. Prove that $m^*(A) = 1$.
- **2.7.** Prove that for any bounded set E, there is a G_{δ} set G for which $E \subset G$ and $m^*(G) = m^*(E)$. Set G is called the *measurable cover* of E. See Theorem 3.1 of the supplemental notes to Section 2.3. (In fact, this result also holds if set E is not bounded, as long as it is of finite measure.)
- **2.8.** Let *B* be the set of rational numbers in the interval [0, 1], and let $\{I_k\}_{k=1}^n$ be a finite collection of open intervals that covers *B*. Prove that $\sum_{k=1}^n m^*(I_k) > 1$. HINT: Mimic the proof of Proposition 2.1 for intervals of the type [a, b].
- **2.9.** Prove that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.