Real Analysis 1, MATH 5210, Fall 2020

Homework 5, 2.3. The σ-Algebra of Lebesgue Measurable Sets, Solutions

Due Sunday, September 27, by noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

2.11. Prove that if a σ-algebra of subsets of $\mathbb{R}$ contains all intervals of the form (a,∞) where $a \in \mathbb{R}$, then it contains all intervals. HINT: There are eight types of intervals: (1) (a,∞), (2) $[a,\infty)$, (3) $-\infty,b)$, (4) $(-\infty,b]$, (5) (a,b), (6) $[a,b)$, (7) $(a,b]$, and (8) $[a,b]$.

2.12. Prove that every interval is a Borel set. HINT: Use Problem 2.11.

2.14. Prove that if a set E has finite positive outer measure, then there is a bounded subset of E that also has positive outer measure. HINT: Consider the contrapositive.

2.3.A. The symmetric difference of sets A and B is $A\Delta B = (A \setminus B) \cup (B \setminus A)$. Prove that if A is measurable and $m^*(A\Delta B) = 0$ then B is measurable and $m^*(A) = m^*(B)$. HINT: Use the fact that the measurable sets form a σ-algebra (by Theorem 2.9) to write B in terms of measurable sets using unions, intersections, and complements. Use the countable additivity of m^* on measurable sets (Proposition 2.13).