Real Analysis 1, MATH 5210, Fall 2020

Homework 6, 2.5. Countable Additivity, Continuity, and the Borel-Cantelli Lemma, 2.6. Nonmeasurable Sets, Solutions

Due Sunday, October 18, by noon

2.28. Prove that continuity of measure together with finite additivity of measure implies countable additivity.

Problem 2.6.A. Show that if $E \in \mathcal{M}$ and $E \subset P$, then $m(E) = 0$. HINT: Let $E_i = E \mathring{+} r_i$, where $\mathbb{Q} \cap [0,1) = \{r_i\}_{i=1}^{\infty}$. Then $\{E_i\}_{i=1}^{\infty}$ is a disjoint sequence of measurable sets and $m(E_i) = m(E)$. Therefore $\sum m(E_i) = m(\cup E_i) \leq m([0,1))$.

Problem 2.6.B. Show that if A is any set with $m^*(A) > 0$, then there is a nonmeasurable set $E \subset A$. HINT: If $A \subset [0,1)$, let $E_i = A \cap P_i$. The measurability of E_i implies $m(E_i) = 0$, while $\sum m^*(E_i) \geq m^*(A) > 0$.

Problem 2.6.C. Give an example $\{E_i\}_{i=1}^{\infty}$ of a disjoint sequence of sets and $m^*(\cup E_i) < \sum m^*(E_i)$. Use set P and explain.