Real Analysis 1, MATH 5210, Fall 2020

Homework 9, 3.1. Sums, Products, and Compositions (of measurable functions)

Due Sunday, November 8, by noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

3.4. Suppose f is a real-valued function defined on $\mathbb{R}$ such that $f^{-1}(\{c\}) \in \mathcal{M}$ for each number c. Is f necessarily measurable? If so, then prove it. If not, then give an example.

3.5. Suppose the function f is defined on a measurable set E and suppose f has the property that $\{x \in E \mid f(x) > c\} \in \mathcal{M}$ for each rational number c. Is f necessarily measurable? If so, then prove it. If not, then give an example.

3.6. Let f be a function with measurable domain D. Prove that f is measurable if and only if the function g defined on $\mathbb{R}$ by $g(x) = f(x)$ for $x \in D$ and $g(x) = 0$ for $x \notin D$ is measurable.

3.7. Let the function f be defined on a measurable set E. Prove that f is measurable if and only if for each Borel set A, $f^{-1}(A)$ is measurable.