Real Analysis 1, MATH 5210, Fall 2024 Homework 2, Section 1.4. Open Sets, Closed Sets, and Borel Sets of Real Numbers Due Saturday, September 7, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). Use the same notation and terminology we used in class and given in the in-class hints.

- **1.4.A.** Prove that a σ -algebra \mathcal{A} of sets of real numbers that contains all intervals of the form [a, b], where a < b, also contains all open sets of real numbers. Does this \mathcal{A} also contain all Borel sets? Why or why not?
- 1.4.B. In mathematical statistics, the set of all possible outcomes of an experiment is called the *sample space* of the experiment. The sample space is required to be a σ -algebra of sets, see my online notes for Intermediate Mathematical Statistics (a slightly lighter version of Mathematical Statistics, MATH 4047/5047) on Section 1.4. Set Theory.

(a) Suppose that the sample space S of some experiment is finite. Prove that the collection of all subsets of S (that is, the power set of S) satisfies the three conditions required to be a collection of events.

(b) Suppose that the sample space S of some experiment is countable. Suppose also that, for every outcome $s \in S$, the subset $\{s\}$ is an event. Prove that every subset of S must be an event.

1.35. Prove that the collection of Borel sets is the smallest σ -algebra that contains the closed sets.